Equations

1 Warm-Up

1. Real numbers x, y satisfy $x^3 - y^2 = 2$. Find $x^9 - 6x^3y^2 - y^6$.
2. Real numbers x, y satisfy $x^3 + y^3 = 9$ and $x^2y + xy^2 = 6$. Find $x + y$.
3. Real numbers a, b, c satisfy $a + b + c = 7$ and $ab + bc + ac = -5$. Find $a^2 + b^2 + c^2$.
4. Real numbers a, b, c satisfy $a^2 + b^2 + c^2 - ab - bc - ca = 0$. Find $a + b - 2c$.
5. Real numbers a, b, c satisfy $a + b + c = 0$. Prove that $a^3 + b^3 + c^3 = 3abc$.
6. Given $a^3 + 7a - 9 = 0$, find $\frac{2a^3+3a}{11a-18}$.
7. Given that $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, prove that $\left(\frac{a+b+c}{b+c+d}\right)^3 = \frac{a}{d}$.

2 Problems

1. Real numbers a, b, c satisfy $\frac{a-c}{b+c} + \frac{b-a}{a+c} + \frac{c-b}{a+b} = 1$. Find
$$\frac{a+b}{b+c} + \frac{b+c}{a+c} + \frac{a+c}{a+b}$$
2. Real numbers a, b, c satisfy $\frac{a}{b+c} = \frac{b}{a+c} = \frac{c}{a+b}$. Find
$$\frac{(a+b)^2}{c^2} + \frac{(a+c)^2}{b^2} + \frac{(b+c)^2}{a^2}$$
3. Real numbers a, b, c satisfy $\frac{1}{a+b+c} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. Prove that
$$\frac{1}{a^3+b^3+c^3} = \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}$$
4. Real numbers a, b, c satisfy
$$\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a} = 0$$
Prove that one of the summands in the left hand side is zero.
5. Real numbers a, b, c satisfy $\frac{1}{a-b} + \frac{1}{b-c} + \frac{1}{c-a} = \frac{3}{2}$. Find
$$\frac{1}{(a-b)^2} + \frac{1}{(b-c)^2} + \frac{1}{(c-a)^2}$$
6. Real numbers a, b, c satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$. Find
$$\frac{ab}{c^2} + \frac{bc}{a^2} + \frac{ca}{b^2}$$
7. Real numbers a, b, c satisfy $\frac{b+c-a}{a} = \frac{a+c-b}{b} = \frac{b+a-c}{c}$. Find
$$\frac{(a+b)(b+c)(c+a)}{abc}$$
8. Given $xyz = 1$, find
$$\frac{1}{1+x+xy} + \frac{1}{1+y+yz} + \frac{1}{1+z+zx}$$