Polynomials

JV Practice 9/15/19

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1 Pre-Problems

1. A quadratic equation $ax^2 - 2ax + b = 0$ has two real solutions. What is the average of these two solutions?
2. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + (1/b)$ and $b + (1/a)$ are the roots of the equation $x^2 - px + q = 0$. What is q?
3. A rectangular box has volume 60 cm^3, surface area 154 cm^2, and its edges have total length 72 cm. Compute the area of the largest face of the box.

2 Problems

1. The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a?
2. Let r, s, and t be the three roots of the equation

$$8x^3 + 1001x + 2008 = 0.$$

   Find $(r + s)^3 + (s + t)^3 + (t + r)^3$.
3. Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?
4. The polynomial $f(x) = ax^3 + bx^2 + cx + d$ has zeros at 1 and -1, and a y-intercept of 2. What is b?
5. Let $g(x) = x^3 - 5x^2 + 2x - 7$, and let the roots of $g(x)$ be p, q, and r. Compute $p^2qr + pq^2r + pqr^2$.
6. Let r_1, r_2, and r_3 be the roots of the polynomial $x^3 - 14x^2 + 15x - 16$. Compute $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.
7. For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

   has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

   What is $f(1)$?

3 Extra Problems

1. Find the integer root of the polynomial $2x^4+7x^3-11x^2+x+1$. For the other three non-integer roots p, q, and r find $pq+qr+pr$.

2. Compute the sum of the roots of the polynomial

$$p(x) = (x^2 - 11x + 1)(x^2 - 11x + 2)(x^2 - 11x + 3) \cdots (x^2 - 11x + 100).$$

3. The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

4. Real numbers r and s are roots of $p(x) = x^3 + ax + b$, and $r + 4$ and $s - 3$ are roots of $q(x) = x^3 + ax + b + 240$. Find the sum of all possible values of $|b|$.

5. Let $f(x) = x^3 + x + 1$. Suppose g is a cubic polynomial such that $g(0) = -1$ and the roots of g are the squares of the roots of f. Find $g(9)$.