Algebra Review

JV Practice 10/13/19
Da Qi Chen

1   Problem Set

1. Given a, b, c are positive reals and $\frac{a+2b}{3a+7b} = \frac{b+2c}{3a+7c}$, find $(a+b)^2 c - (b+c)^2 a$.

2. Consider the function $f(x) = ax^2 + bx + c$. Suppose the sum of the zeros is equal to the product of the zeros and is also equal to the sum of $a + b + c$. Let r be that value. Which of the following is guaranteed to be true: $r = a? r = b? r = c?$

3. Find all roots of $x^3 - 3x^2 + 6x - 4$. Write the roots in both standard form and polar form.

4. Evaluate $\log_3 8 \log_4 5 \log_{25} 17 \log_{17} 9$.

5. Suppose $x^3 + ax^2 + bx + 30$ has three integer roots where $x = 1$ is one of them. What is the smallest value of b?

6. Suppose $3x^2 + ax - 2 = 0$ has only integer roots. What is the sum of all the possible values of a?

7. Suppose $f(x) = x^2 + 6x - 1$ has roots a, b. Suppose $g(x)$ is a cubic function with roots $-a, a^2, b^2$ and $h(x)$ is a cubic with roots $-b, a^2, b^2$. If $f(1)g(1) = 2$, compute $f(2)g(2)$.

8. Let z be a complex number whose conjugate is z'. Find the sum of all z such that $z^2 = z'$.

9. Find all z such that $z^2 = 1 + i$.

10. If $(3 + i)z + 3iz' = 7 + 10i$ where z' is the conjugate of z, then what is z?

11. If $\log_b(a^2) \log_b(c) = 8 \log_a(c)$ and $a - 4b = 0$, find all possible values of a.

12. What is the sum of the base 10 log of all the divisors of 100?

13. Suppose $a_0, a_1, \ldots$ is a geometric sequence where $a_6 = 64$ and the sum of the base 2 log of the first eight terms is 38. What is a_{10}?

14. If a, b, c are reals such that their sum is zero. Find $a^3 + b^3 + c^3 - 3abc$.

15. Given $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$, find $\frac{ab}{c^2} + \frac{bc}{a^2} + \frac{ac}{b^2}$.

16. Give a, b, c are reals and $abc = 1$, find $\frac{1}{1+a+ab} + \frac{1}{1+b+bc} + \frac{1}{1+c+ac}$.

2   Review Quiz

1. Given a, b, c are reals and $\frac{a}{b+c-a} = \frac{b}{a+c-b} = \frac{c}{b+a-c}$, find $\frac{(a+2b)(b+2c)(c+2a)}{abc}$.

2. Suppose a, b, c are real roots of $2x^3 + 3x^2 + 8x + 1 = 0$, what is $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$?

3. If a, b, c are positive integers, find all possible values of c such that $60 = (a + bi)^2 - ci$.

4. If $\log(x^4 y) = 3$ and $\log(\sqrt{x}y^3) = -14$, what is $\log(x^5 y^2)$?