Pascal's Triangle

JV Practice 2/23/20

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1 Warm-Up Problems

1. Prove that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
2. Given $(x+y)^{10}$, find: The coefficient of the x^4y^6 term. The sum of the coefficients.
3. In Pascal's Triangle, each entry is the sum of the two entries above it. In which row of Pascal's Triangle do three consecutive entries occur that are in the ratio $3 : 4 : 5$?

2 Problems

1. Expand $(3x - 2y)^5$.
2. Expand $(t + \frac{1}{t})^7$.
3. Wendy's, a national restaurant chain, offers the following toppings for its hamburgers: {catsup, mustard, mayonnaise, tomato, lettuce, onions, pickle, relish, cheese}. How many different kinds of hamburgers can Wendy's serve, excluding size of hamburger or number of patties?
4. Prove that $\sum_{k=0}^{n} \binom{2n+1}{k} = \frac{1}{2} \sum_{j=0}^{2n+1} \binom{2n+1}{j}$
5. A triangular array of squares has one square in the first row, two in the second, and in general, k squares in the kth row for $1 \leq k \leq 11$. With the exception of the bottom row, each square rests on two squares in the row immediately below (illustrated in given diagram). In each square of the eleventh row, a 0 or a 1 is placed. Numbers are then placed into the other squares, with the entry for each square being the sum of the entries in the two squares below it. For how many initial distributions of 0's and 1's in the bottom row is the number in the top square a multiple of 3?
6. Hockey Stick Identity: prove that $\sum_{k=0}^{m} \binom{n+k}{n} = \binom{n+m+1}{n+1}$