Counting

JV Practice 3/1/20
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1 Warm-Up Problems

1. How many committees of 3 people can be formed from a group of 12 people?

2. (1985 AJHSME) Assume every 7-digit whole number is a possible telephone number except those that begin with 0 or 1. What fraction of telephone numbers begin with 9 and end with 0?

3. (2006 AMC 10A) How many four-digit positive integers have at least one digit that is a 2 or a 3?

4. How many different words can be formed with the letters $AAAABBCCDDDPPP$?

2 Problems

1. (2008 AMC 12B) A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

2. (2005 AMC 10A) How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

3. (2001 AMC 12) A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

4. (2003 AMC 12) How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?

5. (21-228 Bohman) How many positive integers $n < 100$ are not divisible by a square of any integers greater than 1?

6. (2002 AIME) Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left) is $\dfrac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

7. (2013 AIME I) Melinda has three empty boxes and 12 textbooks, three of which are mathematics textbooks. One box will hold any three of her textbooks, one will hold any four of her textbooks, and one will hold any five of her textbooks. If Melinda packs her textbooks into these boxes in random order, the probability that all three mathematics textbooks end up

in the same box can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

8. (2005 AIME II) A game uses a deck of n different cards, where n is an integer and $n \geq 6$. The number of possible sets of 6 cards that can be drawn from the deck is 6 times the number of possible sets of 3 cards that can be drawn. Find n.