Congruence

Varsity Practice
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1 Warm up

1. Segments AB and CD lie on the same line and share same midpoint. Point P is chosen so that APB is isosceles with the base AB. Prove that CPD is also isosceles.

2. (HMMT 2018) How many noncongruent triangles are there with one side of length 20, one side of length 17, and one $60°$ angle?

3. (JBMO 2001 2) Let ABC be a triangle with $\angle C = 90°$ and $CA \neq CB$. Let CL be an interior angle bisector. Show that for $X \neq C$ on the line CL we have $\angle XAC \neq \angle XBC$.

4. On the median BM of a triangle ABC point O is marked so that $\angle OCA = \angle OAC$. Prove that $\triangle ABC$ is isosceles.

5. (SMT 2018 Geo 3) Let ABC be a triangle and D be a point such that A and D are on opposite sides of BC. Given that $\angle ACD = 75^o, AC = 2, BD = \sqrt{6}$, and AD is an angle bisector of both $\triangle ABC$ and $\triangle BCD$, find the area of quadrilateral $ABDC$.

6. (AIME I 2003) Triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 106°$. Point M is in the interior of the triangle so that $\angle MAC = 7°$ and $\angle MCA = 23°$. Find $\angle CMB$.

2 Problems

1. (SMT 2012) Let $ABCD$ be a rectangle with area 2012. There exist points E on AB and F on CD such that $DE = EF = FB$. Diagonal AC intersects DE at X and EF at Y. Compute the area of EXY.

2. (SMT 2012) In quadrilateral $ABCD$, $\angle ABD = \angle BCD$ and $\angle ADB = \angle ABD + \angle BDC$. If $AB = 8$ and $AD = 5$, find BC.

3. (SMT 2018) Let ABC be a right triangle with $\angle ACB = 90°$, $BC = 16$, and $AC = 12$. Let the angle bisectors of $\angle BAC$ and $\angle ABC$ intersect BC and AC at D and E respectively. Let AD and BE intersect at I, and let the circle centered at I passing through C intersect AB at P and Q such that $AQ < AP$. Compute the area of quadrilateral $DPQE$.

4. (AIME I 2003) In convex quadrilateral $ABCD$, $\angle A \cong \angle C$, $AB = CD = 180$, and $AD \neq BC$. The perimeter of $ABCD$ is 640. Find $\cos A$.