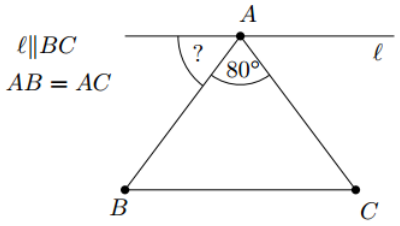
Angle Chasing

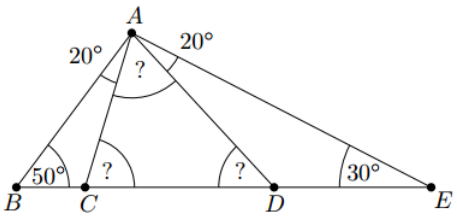
The act of angle chasing is fundamental in geometry, both computational and Olympiad. It is important to recognize when a configuration seems like it can be angle chased, and use this to your advantage.

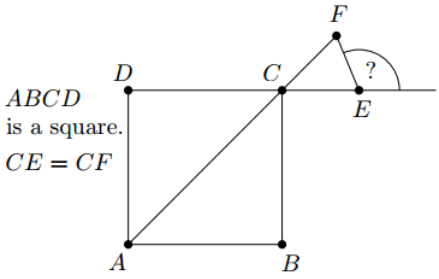
1 Problems

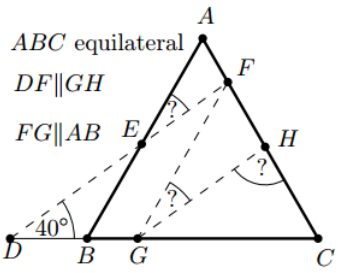
Problems are ordered in terms of difficulty. Challenging problems are marked with a ★.

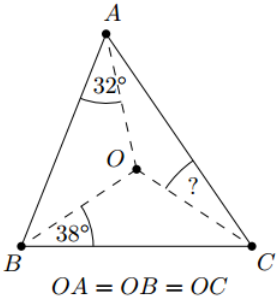
1. In the following problems, find the angles denoted by question marks.

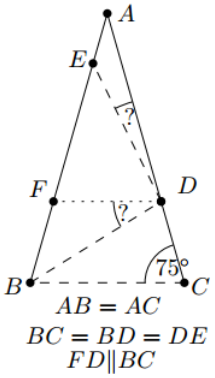










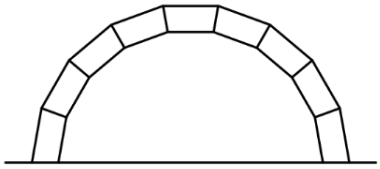


2. [CMIMC 2017, Own] Let ABC be a triangle with $\angle BAC = 117°$. The angle bisector of $\angle ABC$ intersects side AC at D. Suppose $\triangle ABD \sim \triangle ACB$. Compute the measure of $\angle ABC$, in degrees.

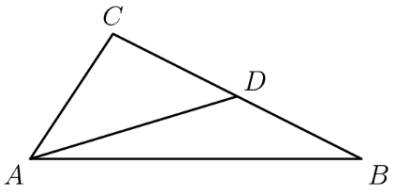
3. [AMSP Team Contest 2015, Own] Suppose $ABCD$ is a convex quadrilateral with $\angle ABC = 144°$, $\angle ADC = 105°$, and $AB = BD = DC$. Compute $\angle BCD - \angle BAD$.

4. [Math League HS 2013-2014/2009-2010/1994-1995] In a certain quadrilateral, the three shortest sides are congruent, and both diagonals are as long as the longest side. What is the degree measure of the largest angle of this quadrilateral?

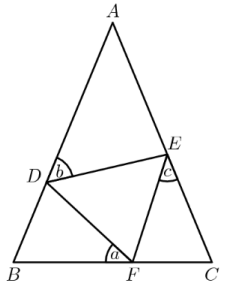
5. [AMC 10B 2009] The keystone arch is an ancient architectural feature. It is composed of congruent isosceles trapezoids fitted together along the non-parallel sides, as shown. The bottom sides of the two end trapezoids are horizontal. In an arch made with 9 trapezoids, let x be the angle measure in degrees of the larger interior angle of the trapezoid. What is x?



6. [AHSME 1957] In triangle ABC, $AC = CD$ and $\angle CAB - \angle ABC = 30^\circ$. What is $\angle BAD$?



7. [AHSME 1960] In this diagram AB and AC are the equal sides of an isosceles triangle ABC, in which is inscribed equilateral triangle DEF. Designate angle BFD by a, angle ADE by b, and angle FEC by c. Then:



(A) $b = \dfrac{a+c}{2}$ **(B)** $b = \dfrac{a-c}{2}$ **(C)** $a = \dfrac{b-c}{2}$
(D) $a = \dfrac{b+c}{2}$ **(E)** none of these

8. [ARML 2006] If $ABCDE$ is a regular pentagon and $MNCD$ is a square (with M and N inside $ABCDE$), compute the value of $m\angle AMN - m\angle EAM$ in degrees.

9. [AIME 2001] In triangle ABC, angles A and B measure 60 degrees and 45 degrees, respectively. The bisector of angle A intersects $\overline{BC}$ at T, and $AT = 24$. Find the area of triangle ABC.

10. [CMIMC 2017, Own] Cyclic quadrilateral $ABCD$ satisfies $\angle ABD = 70^\circ$, $\angle ADB = 50^\circ$, and $BC = CD$. Suppose AB intersects CD at point P, while AD intersects BC at point Q. Compute $\angle APQ - \angle AQP$.

★ 11. [BMO1 1995] Triangle ABC has a right angle at C. The internal bisectors of angles BAC and ABC meet BC and CA at P and Q respectively. The points M and N are the feet of the perpendiculars from P and Q to AB. Find angle MCN.

★ 12. [AMC 10B 2008] Quadrilateral $ABCD$ has $AB = BC = CD$, $\angle ABC = 70^\circ$, and $\angle BCD = 170^\circ$. What is the degree measure of $\angle BAD$?

★ 13. [Naboj 2013] Inside an isosceles triangle ABC fulfilling $AB = AC$ and $\angle BAC = 99.4^\circ$, a point D is given such that $AD = DB$ and $\angle BAD = 19.7^\circ$. Compute $\angle BDC$.