Angles

JV Practice 1/12/20

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Important Principles About Angles

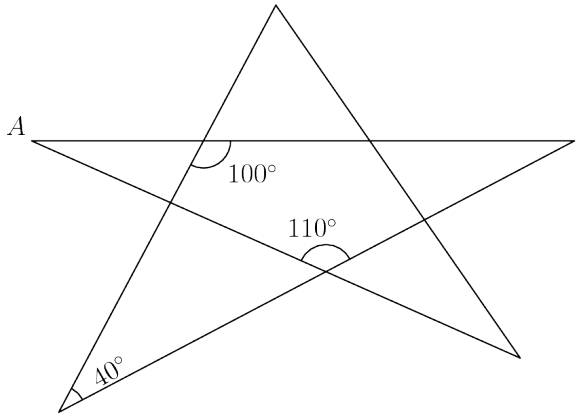
- The angles in a triangle add up to 180 degrees, and in a quadrilateral add up to 360 degrees.
- Complementary angles: Two angles inside a right angle add up to 90 degrees. These two angles are said to be complements of each other.
- Supplementary angles: Two angles on the same side of a straight line add to 180 degrees. These two angles are said to be supplements of each other.
- Opposite/vertical angles: Angles across two intersecting lines are the same.
- Corresponding angles on two parallel lines are the same.
- Exterior angle theorem: The supplementary angle to one angle in a triangle equals the sum of the other two angles in the triangle.
- Inscribed angles (angles formed by two chords of a circle) are half the central angle (angle formed by two radii) of the same arc.
- Inscribed angles are right angles if and only if the opposite chord to the angle is a diameter.

Important Principles That We Will Cover in Less Detail

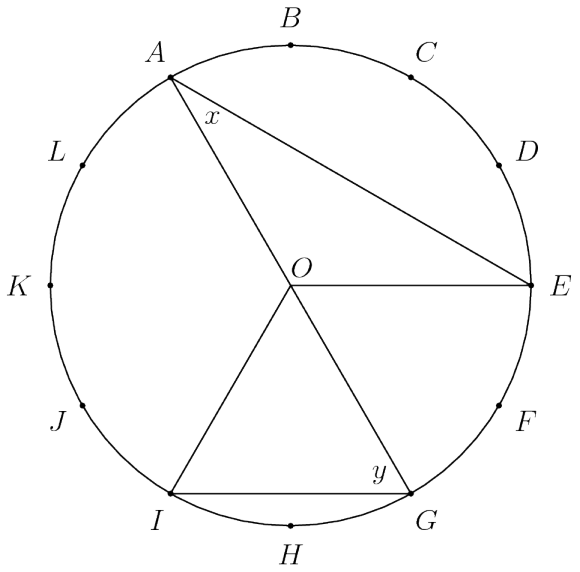
- 360 degrees = 2π radians
- Pythagorean Theorem: $a^2 + b^2 = c^2$ where a, b, and c are the sides of a right triangle and c is the hypotenuse.
- The area of a triangle is $\frac{1}{2}bh$ where b is one side and h is the perpendicular height from that side to the other point.
- The area of a trapezoid is $\frac{b_1+b_2}{2}h$ where b_1 and b_2 are the parallel sides and h is the distance between them.
- There are some popular triangles you may want to get familiar with:
  - 45-45-90 triangles (angles)
  - 30-60-90 triangles (angles)
  - Equilateral triangles
  - Isosceles triangles
  - 3-4-5 triangles (side lengths)
  - Pythagorean triples more generally

Warmup Problems

1. (1999 AMC 8 Problem 21) What is the degree measure of angle A in the following figure?



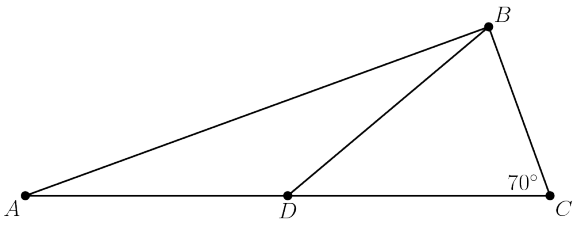
2. (2014 AMC 8 Problem 15) The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?



3. (2016 AMC 12B Problem 4) The ratio of the measures of two acute angles is $5 : 4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

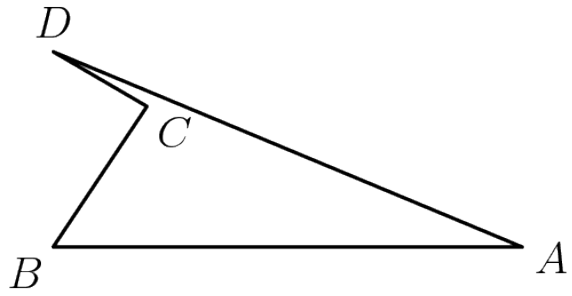
Problems

1. (2014 AMC 8 Problem 9) In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures $70°$. What is the degree measure of $\angle ADB$?

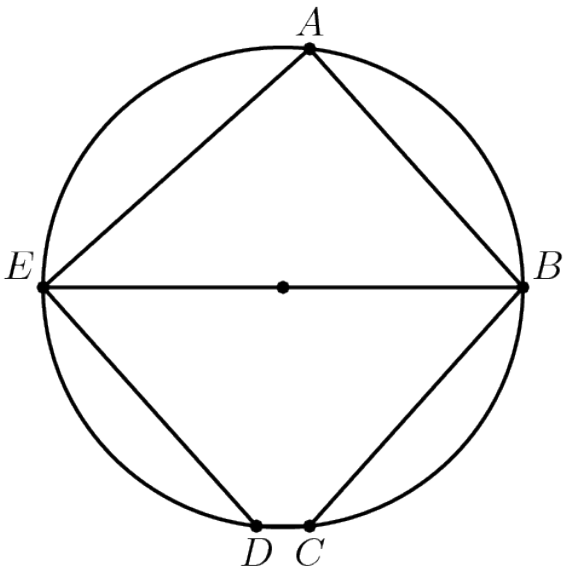


2. (NYCIML Senior F11A23) In $\triangle MAT$, $\overline{AH}$ is a median[1] and $\overline{AH} = \overline{HT}$. Compute the measure of $\angle A$ in degrees.

3. (2017 AMC 8 Problem 18) In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$. What is the area of quadrilateral $ABCD$?



4. In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4 : 5$. What is the degree measure of angle BCD?



5. (NYCIML Senior F10B09) Square $ABCD$ is divided by diagonal $\overline{AC}$. Lines $\overline{BE}$ and $\overline{BF}$ trisect $\angle ABC$ such that point E lies on $\overline{AD}$ and point F lies on $\overline{CD}$. Lines $\overline{EG}$ and $\overline{EH}$

[1]Line from a point that divides the opposite side of the triangle into two equal parts.

trisect $\angle AEB$ such that point G is closer to A on diagonal $\overline{AC}$. Compute the ratio of the measure of $\angle EGH$ to the measure of $\angle EHG$.

6. (2010 AMC 10A Problem 14) Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

7. (2011 AMC 12A Problem 13) Triangle ABC has side-lengths $AB = 12, BC = 24$, and $AC = 18$. The line through the incenter[2] of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

8. (2015 AMC 10A Problem 19) The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

9. (2015 AMC 10B Problem 19) In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

10. (2014 AMC 10A Problem 22) In rectangle $ABCD$, $\overline{AB} = 20$ and $\overline{BC} = 10$. Let E be a point on $\overline{CD}$ such that $\angle CBE = 15^\circ$. What is $\overline{AE}$?

[2]Point where the angle bisectors of a triangle meet.