Area and beyond - part 1

1. Warm-Up

1. *Recall the basics:* The area of triangle is 18 inch^2. Find one of its heights, if one of the sides is equal 6 inch length.

2. The base of the triangle is 4 less than the height, and the area of the triangle is 96. Find the base and the height of the triangle.

3. What is the area of equilateral triangle with the side length 8?

4. Is there exist a triangle with heights of 1, 2, and 3? If so, give an example of such. If not, prove your answer.

5. Inside the parallelogram $ABCD$ an arbitrary point P was selected. The areas of three of the resulting triangles are equal to 1, 2 and 3 (in some order). What values can the area of the fourth triangle take?

6. Find the area of triangle with sides 8, 10, and 12.

7. In trapezoid $ABCD$ ($BC \parallel AD$) the diagonals meet at point O. Prove that the areas of $\triangle AOB$ and $\triangle COD$ are equal.
   Reminder: trapezoid is a quadrilateral with exactly one pair of parallel sides.

2. Problems

1. Prove that if r is a radius of inscribed circle and s is a semi-perimeter ($p = \frac{a+b+c}{3}$, where a, b, and c are the sides of triangle), then the area of this triangle is equal to $\boxed{S_\triangle = sr}$.

2. Prove that for any parallelogram $ABCD$ the areas of $\triangle AOB$, $\triangle BOC$, $\triangle COD$, and $\triangle DOA$ are equal, where point O is the intersection of diagonals: $O = AC \cap BD$.

3. In convex quadrilateral $ABCD$, the diagonals meet at point O. It is known that the areas of $\triangle AOB$ and $\triangle COD$ are equal. Prove that $ABCD$ is a trapezoid or parallelogram.

4. Points M and N on side BC of $\triangle ABC$, and point K on side AC are such that $BM : MN : NC = 1 : 1 : 2$ and $CK : AK = 1 : 4$. It is known that the area of $\triangle ABC$ is 1. Find area of the quadrilateral $AMNK$.

5. Find the area of a parallelogram if one of its sides is 51 and the diagonals are 40 and 74.

6. A line parallel to side AB of $\triangle ABC$ meets side BC at point M and side AC at point N. The ratio of the area of $\triangle MCN$ to the trapezoid $ABMN$ is $4 : 5$. Find $CM : MB$.

7. There is an acute-angled $\triangle ABC$ with $AB = 10$ and $BC = 12$. Let M be the mid-point of side AC in $\triangle ABC$. We know that $BM = 7$. Find the area of $\triangle ABC$?

8. Each diagonal of a convex pentagon $ABCDE$ cuts off a unit area triangle from it. Calculate the area of the pentagon $ABCDE$.

9. Points E and F are the midpoints of sides BC and CD of square $ABCD$. The segments AE and BF intersect at point K. Which figure has a larger area - the $\triangle AKF$ or the quadrangle $KECF$?