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Area and beyond - part 2

1. Warm-Up — *Area and beyond - part 1, Problems Section*

2. Problems

1. The polygon is circumscribed about a circle of radius r. Prove that its area is sr, where s is the semi-perimeter of this polygon.

2. Express the area of trapezoid in terms of it's bases and a height. Prove the correctness of your formula.

3. Prove that medians split a triangle into six triangles with equal areas.

4. Let E and F be the midpoints of the sides BC and AD of the parallelogram $ABCD$. Find the area of the quadrilateral formed by the lines AE, ED, BF, and FC, if the area of $ABCD$ is known to be S.

5. Points M and N are located on the AC side of $\triangle ABC$, and points K and L are on the AB side, with $AM : MN : NC = 1 : 3 : 1$ and $AK = KL = LB$. It is known that the area of $\triangle ABC$ is 1. Find the area of the quadrilateral $KLNM$.

6. Points M and N are dividing side AB of $\triangle ABC$ into three equal parts ($AM = MN = NB$). The points M' and N' are such points on the side BC that $MM' \parallel AC$ and $NN' \parallel AC$. Find the area of the quadrilateral $MNN'M'$, if the area of $\triangle ABC$ is 1.

7. Point A_1 is taken on the side AC of $\triangle ABC$, and point C_1 is taken on the extension of side BC beyond point C. The length of the segment A_1C is 75% of the length of side AC, and the length of segment BC_1 is 120% of the length of side BC. What percentage of the area of $\triangle ABC$ is the area of triangle A_1BC_1?

8. Prove that if in trapezoid $ABCD$ ($BC \parallel AD$) the midpoint M of one side AB is connected to the ends of the other side CD, then the area of the resulting $\triangle CMD$ is half the area of the trapezoid.

9. M is the midpoint of side BC of the unit parallelogram $ABCD$ (area is equal to 1). AM intersects the diagonal BD at point O. Find the area of the quadrilateral $OMCD$.

10. Point E is taken on the side AC of the $\triangle ABC$. D and F are such points on the sides AB and BC that $DE \parallel BC$ and $EF \parallel AB$. Prove that $S_{BDEF} = 2\sqrt{S_{\triangle ADE} \times S_{\triangle EFC}}$.