Invariant

1 Warm-Up

1. In the cow's alphabet there is only two letters: M and O. Their language is quite hard, so in order to simplify it for foreign horses, they make some rules: if you remove two consecutive MO letters, the meaning of the word remains the same. Also, the meaning won't change if you add OM or $MMOO$ in any part of the word. Is it true that MOO and OMM have the same meaning?

2. Is it possible to walk the chessboard with a knight, starting with a1 and ending with h8, and visiting every square exactly once?

3. Alex and Da Qi got the 15×9 milk chocolate with hazelnuts. They take turns choosing one piece of chocolate and braking it along the lines into two pieces. Whoever cannot make the move, loses. Who wins if both Alex and Da Qi make the most optimal moves possible?

4. Numbers $1, \ldots, 6$ are placed on a circle in this order. During one move you can add 1 to any three consecutive ones, or subtract 1 from three numbers that are alternating (no two are consecutive) . Could you make all numbers equal?

2 Problems

1. On the Colorful island there are 3 types of chameleons - red, blue and green. There are 13 red, 15 blue and 17 green. Every time two chameleons of different colors meet, they change their color to a third one (so if red and blue meet, they both become green). Is it possible that at some point all chameleons on the island have the same color?

2. Circle is divided into 6 sectors, each contains a stone. During one move you can pick any two stones and move them to an adjacent sectors to their original positions. Can you gather all stones in one sector?

3. On a 8×8 board in a left bottom 3×3 corner there are 9 stones. During one move you are allowed to take a stone and jump over any other stone on the field that is symmetric to the original field with respect to the second one. Is it possible to collect all stones in the top right 3×3 corner.

4. Circle is divided into 6 sectors which contain numbers $1, 0, 1, 0, 0, 0$. You can add one to any adjacent sectors. Can you make all numbers equal?

5. There are 2000 white balls in the box and an infinite number of white/green and red squares. In one move you can change 2 balls from a box by the following rules: two white or two red for a green one, two green for a white and red, white and red for a green, green and red for a white. After several moves there are only 3 balls left. Prove, that at least one of them is green. Is it possible to leave only one ball in the box?

6. There is a field that has size 10×10 meters, divided into unit 1×1 squares. In the beginning, 9 of these squares are full of weeds. Now, every day all unit squares that are adjacent to at least two other squares full of weeds are also full of weeds by the end of the day. Is it possible that one day the entire field will be full of weeds?