Trigonometry

JV Practice 9/13/20

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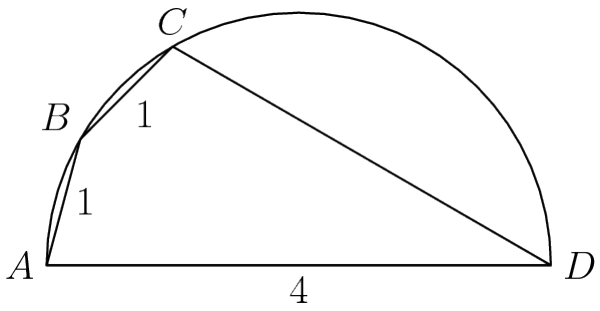
Warm Up Problems

1. (1971 AHSME #31) Quadrilateral $ABCD$ is inscribed in a circle with side AD, a diameter of length 4. If sides AB and BC each have length 1, then what is the length of CD?



2. (2003 AMC 12B #21) An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C. What is the range of α such that $AC < 7$?

3. (Uni of South Carolina 1993 #29) If the sides of a triangle have lengths 2, 3, and 4, what is the radius of the circle circumscribing the triangle?

Guided Problems

1. (1963 AHSME #34) In $\triangle ABC$, side $a = \sqrt{3}$, side $b = \sqrt{3}$, and side $c > 3$. If $\angle C = x$, and x is an integer, what is the smallest possible value of x?

Problems

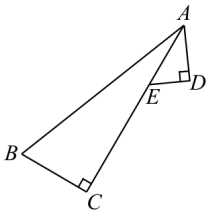
1. (2018 CEMC Fermat #17) Square $PQRS$ has side length 2. Points M and N are the midpoints of SR and RQ, respectively. What is the value of $\cos(\angle MPN)$?

2. (2015 CEMC Euclid #3) $BD = 4$ and point C is the midpoint of BD. If point A is placed so that $\triangle ABC$ is equilateral, what is the length of AD?

3. (2011 CEMC Euclid #4) $ABCD$ is a quadrilateral with $AB = BC = CD = 6$, $\angle ABC = 90°$, and $\angle BCD = 60°$. Determine the length of AD.

4. (2012 CEMC Euclid #5) Triangle ABC has vertices $A(0, 5)$, $B(3, 0)$ and $C(8, 3)$. Determine the measure of $\angle ACB$ in degrees.

5. (2014 CEMC Euclid #8) In the diagram, $\angle ACB = \angle ADE = 90°$. If $AB = 75$, $BC = 21$, $AD = 20$, and $CE = 47$, determine the length of BD.



6. (2019 AMC 10B #16) In $\triangle ABC$ with a right angle at C, point D lies on segment $\overline{AB}$ and point E lies on segment $\overline{BC}$ so that $AC = CD$, $DE = EB$, and the ratio $AC : DE = 4 : 3$. What is the ratio $AD : DB$?

7. (2000 AMC 12 #17) A circle centered at O has radius 1 and contains the point A. The segment AB is tangent to the circle at A and $\angle AOB = \theta$. If point C lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then $OC =$

$$\text{(A) } \sec^2\theta - \tan\theta \qquad \text{(B) } \frac{1}{2} \qquad \text{(C) } \frac{\cos^2\theta}{1+\sin\theta} \qquad \text{(D) } \frac{1}{1+\sin\theta} \qquad \text{(E) } \frac{\sin\theta}{\cos^2\theta}$$

8. (NYCIML Senior B '18-19) In convex pentagon $ABCDE$, $AB = BC = CD = DE = 3$ and $\cos(\angle ABC) = \cos(\angle BCD) = \cos(\angle CDE) = -\frac{1}{3}$. Compute AE^2.

9. (2016 AMC 10A #24) A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of the fourth side?

10. (2013 AIME #9) A paper equilateral triangle ABC has side length 12. The paper triangle is folded so that vertex A touches a point on side $\overline{BC}$ a distance 9 from point B. The length of the line segment along which the triangle is folded can be written as $\frac{m\sqrt{p}}{n}$, where m, n, and p are positive integers, m and n are relatively prime, and p is not divisible by the square of any prime. Find $m + n + p$.

11. (1989 AIME #10) Let a, b, c be the three sides of a triangle, and let α, β, γ, be the angles opposite them. If $a^2 + b^2 = 1989c^2$, find $\frac{\cot\gamma}{\cot\alpha + \cot\beta}$