Logarithms

JV Practice

1 Warmup

1. What is $b^x \cdot b^y$?
2. What is $\frac{b^x}{b^y}$?
3. What is $(b^x)^y$?
4. If $b \neq 0$, what is b^0?

2 Log Rules

- Definition: If $b^x = a$ where $b > 0$ and $x > 0$, then $x = \log_b(a)$.
- Multiplication: $\log_b(xy) = \log_b(x) + \log_b(y)$
- Division: $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$
- Exponentiation: $\log_b(x^y) = y \log_b(x)$
- Convention: Usually, $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- For any $b \neq 0$, $\log_b(1) = 0$ because $b^0 = 1$, and $\log_b(b) = 1$ because $b^1 = b$.
- $b^{\log_b(a)} = a$

3 Examples

1. $\log_2(16) = 4$ because $2^4 = 16$
2. $\log_5(125) = 3$ because $5^3 = 125$
3. $\log_2(\frac{1}{2}) = -1$ because $2^{-1} = \frac{1}{2}$
4. $\log_8(2) = \frac{1}{3}$ because $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

4 Problems 1

Find all solutions to the following equations:

1. $\log_4\left(x^2 - 2x\right) = \log_4(5x - 12)$
2. $\ln(x) + \ln(x+3) = \ln(20 - 5x)$
3. $\log_2(x+5) - \log_2(2x-1) = 5$
4. $3^x = 7^{4x+2}$

5 Another Log Rule

- Change of Base: $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$ (the **b**ase goes on the **b**ottom)
- Fun identity (follows from Change of Base): $\log_a(b) = \frac{1}{\log_b(a)}$

6 Problems 2

1. (2003 AMC 12B Problem 17) If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?
2. (2005 AMC 10B Problem 17) Suppose that $4^a = 5, 5^b = 6, 6^c = 7$, and $7^d = 8$. What is $a \cdot b \cdot c \cdot d$?
3. (2010 AMC 12A Problem 11) The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

7 Problems 3

1. (NYCIML F10B25) Compute $(\log_{125} 16)(\log_4 27)(\log_3 625)$.
2. (NYCIML F06B07) Compute
$$\frac{\log 8}{\log \frac{1}{8}}$$
3. (NYCIML S11B26) Let $\log_{10} 70 = m$ and $\log_{10} 20 = p$. Given that $\log_{10} 14 = Am + Bp + C$ where A, B, and C are integers, compute the ordered triple (A, B, C).
4. (NYCIML F06A19) If $\log_b(a)\log_c(a)\log_c(b) = 25$ and $\frac{a^2}{c^2} = c^k$, what is the sum of all possible values of k?

8 Challenge Problems

1. (2000 AIME II Problem 1) The number $\frac{2}{\log_4 2000^6} + \frac{3}{\log_5 2000^6}$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

2. The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}} x))))$ is an interval of length $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?