Euclidean Algorithm

JV Practice 11/3/19

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1 Pre-Problems

Do the following problems **without using a calculator**.

1. Find the greater common divisor (gcd) of 7544 and 115.
2. Find integers x and y such that $7544x + 115y = gcd(7544, 115)$
3. What is the least common multiple of 7544 and 115?

2 Problems

1. What is the greatest common divisor (gcd) of 11571 and 1767?
2. (AMC 8 2013) What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?
3. Find the greatest common divisor d of 143 and 26, and find integers x and y solving the equation $143x + 26y = d$.
4. Prove that numbers $27x + 4$ and $18x + 3$ are coprime for any integer x.
5. (1986 AIME Problem 5) What is the largest positive integer n such that $n^3 + 100$ is divisible by $n + 10$?
6. Find the greatest common divisor of $x^4 + x^3 - 4x^2 + x + 5$ and $x^3 + x^2 - 9x - 9$.
7. What is the sum of all integer n such that $n^2 + 2n + 2$ divides $n^3 + 4n^2 + 4n - 14$?
8. (1985 AIME Problem 13) The numbers in the sequence 101, 104, 109, 116,... are of the form $a_n = 100 + n^2$, where $n = 1, 2, 3, \ldots$ For each n, let d_n be the greatest common divisor of a_n and a_{n+1}. Find the maximum value of d_n as n ranges through the positive integers.
9. (AMC 8 2016) The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?