Fermat's Little Theorem

JV Practice 11/24/19

C.J. Argue

Pre-Problems

1. Compute $3^{31} \pmod 7$.
2. Find the remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ is divided by 7.
3. Find the smallest positive integer x such that $x^{103} \equiv 4 \pmod{11}$.

Round 1

1. [1 pt] Compute $10^{73} \pmod{19}$.
2. [1 pt] Find the two smallest integers x such that $x^{86} \equiv 6 \pmod{29}$.
3. [2 pts] Compute $2^{98} \pmod{33}$.

Round 2

1. [2 pts] If a googolplex is $10^{10^{100}}$, what day of the week will it be a googolplex days from now? (Today is Sunday.)
2. [3 pts] Find all prime numbers p such that $29^p + 1$ is a multiple of p.
3. [3 pts] The sequence
$$x, x^2, x^3, \ldots \pmod{13}$$
is periodic for every integer value of x. List all possible periods this sequence could have.

Round 3

1. [3 pts] Find $3^{1000000} \pmod{19}$.
2. [4 pts] Find all positive integers x such that $2^{2^x+1} + 2$ is divisible by 17.
3. [4 pts] Find the smallest prime number that does not divide $9 + 9^2 + 9^3 + \cdots + 9^{2010}$.
4. [5 pts] If $f(x) = x^{x^{x^x}}$, find $f(17) \pmod{92}$.