Diophantine equations

If nothing stated, then find all positive integer solutions of the given equation.

1 Problems to think about

1. Find all positive integers x, y such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{31}$.
2. Find all integers x, y such that $x^3+y^3=(x+y)^2$.
3. Find all positive integers x, y such that $x^5-y^2=4$.

2 Warm-Up

1. $(x^2+1)(y^2+1)+2(x-y)(1-xy)=4(1+xy)$
2. $x^2+y^2+z^2+2xy+2x(z-1)+2y(z+1)=w^2$
3. $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$
4. $(x+1)^2+\ldots+(x+2001)^2=y^2$
5. $x^3+2y^3=4z^3$
6. Prove that there are no positive integers x, k and $n \geq 2$ such that $x^2+1=k(2^n-1)$.

3 Problems

1. $x^3+y^3+z^3-3xyz=101$.
2. $x^3+(x+1)^3+\ldots+(x+7)^3=y^3$ in $\mathbb{Z}$
3. Prove, that $x^3+y^3+z^3=x^2+y^2+z^2$ has infinite number of solutions in $\mathbb{Z}$
4. $5^x 7^y+4=3^z$
5. $4xy-x-y=z^2$
6. Let a and b be positive integers such that $ab+1$ divides a^2+b^2. Prove that $\frac{a^2+b^2}{ab+1}$ is a perfect square.