Bases

JV Practice 6/14/20

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Warmup

In the following, we write $\overline{a_0a_1a_2\ldots a_n}_b$ to represent $\overline{a_0a_1a_2\ldots a_n}$ written in base b. We usually use base 10 so in the following examples I will not put a subscript if the number is represented in base 10. If you haven't seen this before, try using these examples to help you do the warmup:

$$251_8 = 2\cdot 8^2 + 5\cdot 8 + 1\cdot 8^0 = 128 + 40 = 169 = 3\cdot 7^2 + 3\cdot 7 + 1\cdot 7^0 = 331_7$$

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1. Convert 10_7 into base 10.
2. Convert 321_9 into base 10.
3. Convert 284 base 10 into base 6.
4. Convert $1a_{11}$ to base 10.
5. Convert 111011010000_2 into base 8.

Problems

1. Express $1101_2 \times 1111_2$ in base 10.
2. Express 0.25 in base 10 as a decimal in base 2.
3. Express abc_{16} in base 10.
4. Express $\frac{8}{23}$ in base 10 as a number in base 7.
5. A rational number written in base eight is $\underline{ab}.\underline{cd}$, where all digits are nonzero. The same number in base twelve is $\underline{bb}.\underline{ba}$. Find the base-ten number $\underline{abc}$.
6. Find the value of the base b such that the following addition problem is correct:
$$6651_b + 115_b = 10066_b$$
7. Find the sum of all the natural numbers that are three-digit palindromes when expressed in base 5. Express your answer in base 5.
8. How many of the numbers $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{30}$ have non-terminating expansions in base 30?
9. (a) How many 0s are at the end of 15! when written in base 12?
   (b) How many 0s are at the end of 100! when written in base 24?

10. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3

11. For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323...{}_k$. What is k?

12. What is the largest positive integer n less than $10,000$ such that in base 4, n and $3n$ have the same number of digits; in base 8, n and $7n$ have the same number of digits; and in base 16, n and $15n$ have the same number of digits? Express your answer in base 10.