Exponents

JV Practice 6/14/20

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Warmup

1. Find n such that

$$2^n = \frac{(2^2 \cdot 2^3)^2}{4^{-3/2} \cdot 64^{2/3}}$$

2. Find the remainder when
   (a) 3^3 is divided by 4
   (b) 3^4 is divided by 4
   (c) 3^{120} is divided by 4.

3. Find all values of x such that $5^x = 125$.

Problems

1. Given that $3^{2x+4} = 9^{2x-6}$ what are the possible values of x?

2. Given that $2^x = 12$ find 2^{2x-1}

3. Find the units digit of 235413^{235}

4. What is the smallest positive integer x such that 2^x is greater than five million?

5. Find the value of

$$\frac{2^{2004} + 2^{2001}}{2^{2003} - 2^{2000}}$$

6. Find the value of x that satisfies the equation

$$25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$$

7. Let m be the number of digits in 2^{2006} and n be the number of digits in 5^{2006}. Find $m + n$.

8. Given that $3^8 \cdot 5^2 = a^b$ where both a and b are positive integers. Find the minimum possible value of $a + b$.

9. What is the minimum number of digits to the right of the decimal point needed to express the fraction

$$\frac{123456789}{2^4 \cdot 5^{26}}$$

as a decimal?

10. Determine the smallest element in the set

$$S = \left\{ \left(\frac{1}{2}\right)^{1/2}, \left(\frac{1}{3}\right)^{1/3}, \left(\frac{1}{4}\right)^{1/4}, \left(\frac{1}{5}\right)^{1/5}, \left(\frac{1}{6}\right)^{1/6} \right\}.$$

11. Let the sequence $\{x_n\}$ be defined as $x_1 \in \{5, 7\}$ and for $k \geq 1, x_{k+1} \in \{5^{x_k}, 7^{x_k}\}$. For example, all the possible value of x_3 are $5^{5^5}, 5^{5^7}, 5^{7^5}, 5^{7^7}, 7^{5^5}, 7^{5^7}, 7^{7^5}, 7^{7^7}$. Determine the sum of all possible values of the last two digits of x_{2012}

12. Suppose that $60^a = 3$ and $60^b = 5$. Compute the value of $12^{\frac{1-a-b}{2-2b}}$.

13. Find all ordered pairs (x, y) of real numbers such that

$$3^{x^2-2xy} = 1 \quad \text{and} \quad x^2 = y + 3$$

14. Compute all real numbers x such that $\sqrt[3]{8+x} + \sqrt[3]{8-x} = 1$

15. If k and n are integers and $(3^{2006} + 2006)^2 - (3^{2006} - 2006)^2 = k \cdot 3^n$, where k is not divisible by 3, compute $\frac{n+k}{2006}$

Only do the next section if you finish all the previous problems!

Review/Extensions

1. The number n can be written in base 14 as $\underline{abc}$, can be written in base 15 as $\underline{acb}$, and can be written in base 6 as $\underline{acac}$, where $a > 0$. Find the base 10 representation of n.

2. Find the number of positive integers m for which there exist nonnegative integers $x_0, x_1, x_2, \ldots, x_{2011}$ such that

$$m^{x_0} = \sum_{k=1}^{2011} m^{x_k}$$

3. Find the number of permutations $(a_1, a_2, \ldots, a_{30})$ of $1, 2, \ldots, 30$ such that for $m \in \{2, 3, 5\}$, m divides $a_{n+m} - a_n$ for all integers n with $1 \leq n < n + m \leq 30$.