Modular Arithmetic

V Practice 7/19/20

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1 Modular Arithmetic

1. Compute 2019^{2018} mod 2020.
2. Compute x such that $23x \equiv 1 \bmod 41$.
3. Compute 33! mod 37.
4. Find the smallest positive integer n such that 107n has the same last two digits as n.
5. Compute the largest integer that has the same number of digits when written in base 5 and when written in base 7. Express your answer in base 10.
6. Compute 2018^{2019} mod 2020.
7. What are the last 8 digits of $11 * 101 * 1001 * 10001 * 100001 * 1000001 * 111$?
8. Define the sequence a_n via $a_1 = 7$ and $a_k = 7^{a_{k-1}}$. Find the last two digits of a_{2007}.

2 Modular Polynomials

1. Let $f(x, y) = x^2 + y^2$. For how many pairs (x, y) where $x, y \in [0, 30]$ does $f(x, y) \equiv 5 \bmod 9$?
2. Is 23 a square mod 41? Is 15 a square mod 41?
3. What's the smallest $n > 1$ such that $n^3 \equiv n \bmod 1000$? What if we wanted instead $n^2 \equiv n$?
4. Let $f(x) = \sum_{i=1}^{7} x^i$. Let $S = f(7) + f(11) + f(13)$. Compute S mod 17.
5. Find the largest integer n such that $7^{2048} - 1$ is divisible by 2^n.
6. Let $f(x) = \sum_{i=1}^{6} ix^i$. Let $S = [f(6)]^5 + [f(10)]^3 + [f(15)]^2$. Compute S mod 30.