Ratios of Area

Varsity Practice 4/18/21

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1 Warm-Up

1. (AMC 10 2020) Triangle AMC is isosceles with $AM = AC$. Medians $\overline{MV}$ and $\overline{CU}$ are perpendicular to each other, and $MV = CU = 12$. What is the area of $\triangle AMC$?

2. (HMMT) A rectangle is folded along its diagonal to form a nonconvex pentagon. The area of this pentagon is $\frac{7}{10}$ the area of the original rectangle. If the rectangle has sides of length a, b with $a > b$, compute $\frac{a}{b}$.

2 Challenge Problems

1. (HMMT 2014) Let ABC be a triangle with sides $AB = 6, BC = 10$, and $CA = 8$. Let M and N be the midpoints of BA and BC, respectively. Choose the point Y on ray CM so that the circumcircle of triangle AMY is tangent to AN. Find $[NAY]$.

2. (HMMT 2010) Let O be the point $(0, 0)$. Let A, B, C be three points in the plane such that $AO = 15$, $BO = 15$, and $CO = 7$, and such that $[ABC]$ is maximal. What is the length of the shortest side of ABC? *Hint: do geometry first, then algebra.*