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Very inscribed angles - part 1

1. Warm-Up

1. a) Let ABC be an arbitrary triangle. Let I be its incenter and let D be the point where line BI crosses the circumcircle of $\triangle ABC$. Prove that D is equidistant from A, C, and I.

   b) Prove that the point symmetric to the orthocenter of a triangle with respect to its side lies on the circumscribed circle of this triangle.

2. The bisector of the outer angle at the vertex C of $\triangle ABC$ intersects the circumcircle at point D. Prove that $AD = BD$.

3. Points A, B, C and D lie on a circle. Points M, N, K and L are the midpoints of arcs AB, BC, CD and DA, sequentially located on a circle. Prove that chords MK and NL are perpendicular.

4. On the side BC of triangle ABC, as on the diameter, a circle is constructed that intersects the segment AB at point D. Find the ratio of the areas of $\triangle ABC$ and $\triangle BCD$ if you know that $AC = 15, BC = 20$ and $\angle ABC = \angle ACD$.

5. Point O is the center of the circumscribed circle of an acute-angled $\triangle ABC$. The altitude AH is drawn from the vertex A. Prove that $\angle BAH = \angle OAC$.

6. The altitude AH is drawn in $\triangle ABC$; O is the center of the circumscribed circle. Prove that $\angle OAH = |\angle B - \angle C|$.

7. A circle with the center O inscribed in quadrilateral $ABCD$ and touches its non-parallel sides BC and AD in points E and F. Let the line AO and segment $\overline{EF}$ intersect at point N, and lines BK and CN at point M. Prove that points O, K, M, and N lie on the same circle.

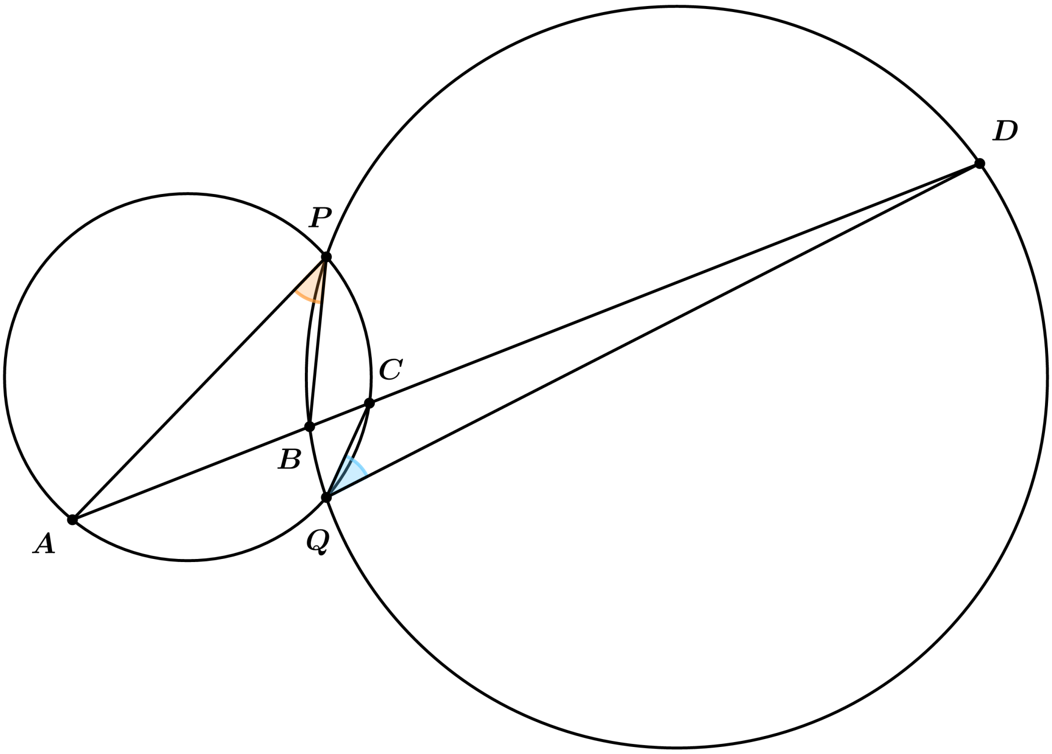
2. Problems

1. A circle is constructed on the leg AC of a right-angled $\triangle ABC$ as on a diameter, intersecting the hypotenuse AB at point K. Find CK if $AC = 2$ and $\angle A = 30°$.

2. From an arbitrary point M, lying inside of a given angle with apex A, perpendiculars MP and MQ are dropped on the sides of the angle. Perpendicular AK is dropped from the point A to a segment PQ. Prove that $\angle PAK = \angle MAQ$.

3. A circle s with center O and a circle s' intersect in points A and B. On the arc of circle s that lie inside the circle s' point C was chosen. Let the intersection points of AB and BC with s' other than A and B will be E and D respectively. Prove that the lines DE and OC are perpendicular.

4. Two circles intersect in two points P and Q. A line intersect these two circles in four points A, B, C and D like in is shown in the diagram below. Prove that $\angle APB = \angle CQD$.



5. In acute-angled $\triangle ABC$ point O is a center of its circumcircle. Through the points O, B and C were circumscribed a circle s. Let OK be a diameter of the circle s, as well as points D and E be the points of its intersection with lines AB and AC respectively. Prove that $ADKE$ is a parallelogram.

6. Points D, E and F are taken on the sides AB, BC and AC of $\triangle ABC$, respectively, so that $DE = BE$ and $FE = CE$. Prove that the circumcenter of $\triangle ADF$ lies on the bisector of $\angle DEF$.

7. Inside the parallelogram $ABCD$ was chosen such point M, as well as inside $\triangle AMD$ point N that $\angle MNA + \angle MCB = \angle MND + \angle MBC = 180°$. Prove that lines MN and AB are parallel.

8. A circle with center O inscribed in $\triangle ABC$ and tangent its sides AB, BC, and AC in points E, F, and D respectively. Lines AO and CO intersect line EF in points N and M. Prove that the circumcenter of $\triangle OMN$, point O and D lie in one line.

3. Bonus

1. The diagonals of the inscribed quadrilateral $ABCD$ meet at point M, $\angle AMB = 60°$. Equilateral triangles ADK and BCL are built on the sides AD and BC outside of $ABCD$. Line KL meets the circumcircle of $ABCD$ at points P and Q. Prove that $PK = LQ$.