Very inscribed angles - part 2

1. From the previous series...

1. Prove that the point symmetric to the orthocenter of a triangle with respect to its side lies on the circumscribed circle of this triangle.
   Last time we proved this fact for any acute-angled triangle. Now, prove it for any obtuse-angled triangle.

2. Let points A, B, C and D lie on the circle in this order, and point M is an intersection of continuation of lines AB and CD. Prove that $\angle AMD$ is equal to half difference between arcs AD and BC.

3. AH is an altitude in $\triangle ABC$; O is its circumcenter. Prove that $\angle OAH = |\angle B - \angle C|$.
   Last time we proved this fact for any acute-angled triangle. Now, prove it for any obtuse-angled triangle.

2. Problems

1. Point D is an arbitrary point on the side AB of isosceles $\triangle ABC$ where $AB = BC$. Triangles ADC and BDC were inscribed in the circles s_1 and s_2. The tangent to the circle s_1 in point D intersect the circle s_2 at M other than point D. Prove that $BM || AC$.

2. Circles s_1 and s_2 with centers O_1 and O_2 intersect at A and B. Ray O_1B intersect s_2 at F, and ray O_2B intersect s_1 at E. Line that goes through the point B parallel to the line EF intersects circles s_1 and s_2 second time at M and N respectively. Prove that $MN = AE + AF$.

3. Points O_1 and O_2 are circumcenter and incenter of isosceles $\triangle ABC$ ($AB = BC$). Circumcircles of $\triangle ABC$ and $\triangle O_1O_2A$ intersect at A and D. Prove that the line BD tangent to circumcircle of $\triangle O_1O_2A$.

4. On the side AC in $\triangle ABC$ points D and E were found such as $AB = AD$ and $BE = EC$ (point E is in between points A and D). Point F is a middle of arc BC of circumcircle of $\triangle ABC$. Prove that points B, E, D, F lie on one circle.

5. Point D is an arbitrary point on the side AB of $\triangle ABC$. Circumcircle of $\triangle BCD$ intersect side AC at M, and circumcircle of $\triangle ACD$ intersect side BC at N ($M, N \neq C$). Let point O be a circumcenter of $\triangle CMN$. Prove that the line OD is perpendicular to the side AB.

6. In $\triangle ABC$ ($AB > BC$) points K and M are midpoints of sides AB and AC, point O is incenter of $\triangle ABC$. Let point P be an intersection of lines KM and CO, and point Q such that $QP \perp KM$ and $QM || BO$. Prove that $QO \perp AC$.

3. Bonus

1. We have a convex quadrilateral $ABCD$. Let l_A, l_B, l_C, and l_D be bisectors of external angles of our quadrilateral. Let $K = l_A \cap l_B$, $L = l_B \cap l_C$, $M = l_C \cap l_D$, and $N = l_D \cap l_A$. Prove that if the circumcircles of $\triangle ABK$ and $\triangle CDM$ are externally tangent, then the circumcircles of $\triangle BCL$ and $\triangle DAN$ are also externally tangent.