Geometry Algebra Style

Varsity Practice 10/25/20

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1 Warm-Up Problems

1. (1999 AHSME #15) Let x be a real number such that $\sec x - \tan x = 2$. Compute $\sec x + \tan x$.
2. (2007 AMC 12A #17) Suppose that $\sin a + \sin b = \sqrt{\frac{5}{3}}$ and $\cos a + \cos b = 1$. What is $\cos(a-b)$?
3. (1996 AIME #10) Find the smallest positive integer solution to $\tan 19x^\circ = \dfrac{\cos 96^\circ + \sin 96^\circ}{\cos 96^\circ - \sin 96^\circ}$.

2 Problem Set

1. (1999 AHSME #27) In triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then $\angle C$ is how many degrees?
2. (2006 iTest #17) Let $\sin(2x) = \frac{1}{7}$. Find the numerical value of $\sin(x)\sin(x)\sin(x)\sin(x) + \cos(x)\cos(x)\cos(x)\cos(x)$.
3. (2008 iTest #57) Let a and b be the two possible values of $\tan\theta$ given that $\sin\theta + \cos\theta = \dfrac{193}{137}$. If $a+b = m/n$, where m and n are relatively prime positive integers, compute $m+n$.
4. (1995 AIME #9) Triangle $\overline{ABC}$ is isosceles, with $AB = AC$ and altitude $AM = 11$. Suppose that there is a point D on $\overline{AM}$ with $AD = 10$ and $\angle BDC = 3\angle BAC$. Then the perimeter of $\triangle ABC$ may be written in the form $a + \sqrt{b}$, where a and b are integers. Find $a+b$.
5. (1995 AIME #7) Given that $(1+\sin t)(1+\cos t) = 5/4$ and $(1-\sin t)(1-\cos t) = \frac{m}{n} - \sqrt{k}$, where k, m, and n are positive integers with m and n relatively prime, find $k+m+n$.
6. (1991 AIME #9) Suppose that $\sec x + \tan x = \frac{22}{7}$ and that $\csc x + \cot x = \frac{m}{n}$, where $\frac{m}{n}$ is in lowest terms. Find $m+n$.
7. (2003 AIME I #4) Given that $\log_{10}\sin x + \log_{10}\cos x = -1$ and that $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, find n.
8. (2008 AIME I #8) Find the positive integer n such that
$$\arctan\frac{1}{3} + \arctan\frac{1}{4} + \arctan\frac{1}{5} + \arctan\frac{1}{n} = \frac{\pi}{4}.$$
9. (2006 AIME I #12) Find the sum of the values of x such that $\cos^3 3x + \cos^3 5x = 8\cos^3 4x \cos^3 x$, where x is measured in degrees and $100 < x < 200$.
10. (1980 USAMO #3) $A+B+C$ is an integral multiple of π. x, y, and z are real numbers. If $x\sin(A) + y\sin(B) + z\sin(C) = x^2\sin(2A) + y^2\sin(2B) + z^2\sin(2C) = 0$, show that $x^n\sin(nA) + y^n\sin(nB) + z^n\sin(nC) = 0$ for any positive integer n.